# FLOW MEASUREMENT ORIFICE PLATE SIZING AND UNCERTAINTY ANALYSIS

This case study demonstrates the use of some of Flownex's power features – the Designer and the Sensitivity Analysis capability – during the design and uncertainty evaluation of flow measurement using an orifice plate connected to a pressure transducer and transmitter.

OIL AND GAS INDUSTRY

## CHALLENGE:

The main challenge for this case study is the application of Flownex to:

- Size an orifice plate to be used in conjunction with a flow transmitter to serve as an accurate flow meter for natural gas.
- Evaluate the measurement uncertainty of the flow meter with variations in operating conditions and manufacturing tolerances.

## BENEFITS:

Flownex is an ideal tool to design gas flow systems, including piping, valving and most other components that are typically found in the oil and gas industry. Not only is Flownex also the ideal tool to design accurate flow measurement orifice plates, but it also has the capability to evaluate the uncertainty of the flow meter in general and the orifice plate in particular with the inevitable variations in conditions and manufacturing tolerances.

## SOLUTION:

Using the *Designer* and the *Sensitivity Analysis* features built into Flownex, the orifice plate can be designed and its operational uncertainty evaluated when functioning in combination with the pressure transducer/flow transmitter.

"Not only is Flownex capable of designing large, sophisticated flow networks, but it is also able to focus on a single element of design such as a simple orifice plate and then analyse that element in incredible depth. This versatility of Flownex is unparalleled amongst similar products and must make Flownex an indispensable design software tool amongst process design engineers."

*Hannes van der Walt*
*Principal Thermal Engineer*
*Gasco Pty Ltd*

## INTRODUCTION

Flow transmitters use a variety of measurement techniques, the simplest (and cheapest) of which is probably the orifice plate. These orifice plates are often designed in accordance with ISO 5167 or BS 1042. The pressure transducers used by flow transmitters are typically calibrated by mapping the orifice plate minimum to maximum measured pressure difference, to the typical 4-20 mA signals they transmit. At zero flow the transmitter would issue a 4 mA signal. As a rule of thumb, the minimum actual flow would be calibrated to approximately 30% above the minimum signal (i.e. 8.8 mA) whereas the maximum flow would be calibrated to approximately 70% to 80% of full scale (14 to 16 mA).

However, when such an orifice plate is sized, the sizing calculation has to be based on one specific operational point - one pressure, one temperature and one flow rate - resulting in one fixed orifice plate size. In practice, however, the pressure and temperature will not remain constant and neither will the flow rate. Furthermore, the physical orifice plate has to be manufactured and the manufacturing tolerance will also have an influence on the measurement uncertainty of the orifice plate. According to ISO 5167 there is also an uncertainty associated with the calculated discharge coefficient which is directly proportional to the discharge mass flow rate calculated for the orifice plate. This uncertainty is also discussed later in this case study.

"Flownex has a very powerful design facility in terms of the *Designer* feature. The *Designer* is typically used to "goal seek" for a solution. Flownex also offers the ability to perform Monte Carlo analyses where multiple independent variables are randomly varied such that the maximum and minimum combined influence of the independent variables may be observed on the dependent variables."

Flownex has a very powerful design facility in terms of the *Designer* feature. The *Designer* is typically used to "goal seek" for a solution, and in this case study the *Designer* has been used to find the required orifice plate size to comply with functional requirements. The measurement uncertainty of the orifice plate is evaluated by using the *Sensitivity Analysis* feature of Flownex. This feature allows for parametric studies where a single independent variable is varied to study its influence on a single or multiple dependent variables. Flownex also offers the ability to perform Monte Carlo analyses where multiple independent variables are randomly varied such that the maximum and minimum combined influence of the independent variables may be observed on the dependent variables.

# Flownex Model

Figure 1 shows the example network with an orifice plate between two pipes. The link between the upstream pipe and the orifice plate element is used to transfer the pipe internal diameter to the orifice plate element.

For the case at hand, the design information and requirements are as follows:

- Maximum fuel gas flow rate = 1500 $Nm^3$/hr.
- Nominal fuel gas flow rate = 1200 $Nm^3$/hr.
- Minimum fuel gas flow rate = 350 $Nm^3$/hr.
- Maximum temperature = 30°C.
- Nominal temperature = 25°C.
- Minimum temperature = 5°C.
- Supply pressure = 120 kPa-g +3.7 kPa -6.3 kPa.
- Target pressure drop at maximum flow = 5 kPa.
- Target pressure drop at minimum flow = 0.5 kPa.
- Transmitter accuracy = ±0.075% of span (from manufacturer).
- Transmitter repeatability/stability = ±0.01% of span (from manufacturer).
- Orifice plate diameter manufacturing tolerance = ±0.1 mm.

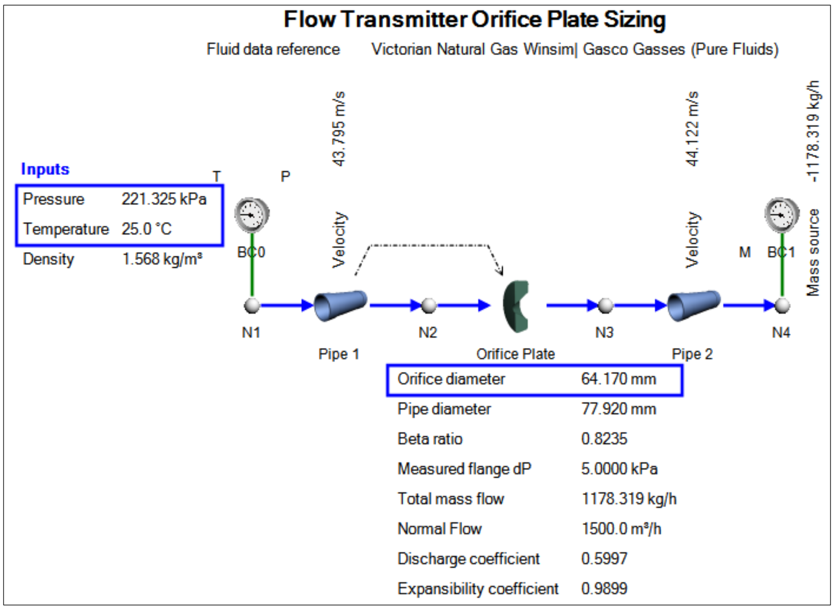



*Figure 1: Example Flownex Orifice Plate Pipe Network.*

The nominal operating upstream pressure and temperature are specified at the inlet boundary whilst the required flow rate is specified as a sink (negative mass source) at the outlet boundary.

# Sizing the Orifice Plate

Flownex uses the BS 1042 formulation to calculate the orifice plate measured pressure difference which is equivalent to the ISO 5167 correlation. Refer to the Flownex Library help on restrictors for the

Flownex calculation method. However, the ISO 5167 correlation is more comprehensive and also provides methods to estimate flow rate uncertainties. According to ISO 5167 an orifice plate discharge may be calculated as follows:

$$\dot{\boldsymbol{m}} = \frac{\boldsymbol{C}}{\sqrt{\boldsymbol{1}-\boldsymbol{\beta}^{\boldsymbol{4}}}}\boldsymbol{\varepsilon}\frac{\boldsymbol{\pi}}{\boldsymbol{4}}\boldsymbol{d}^{\boldsymbol{2}}\sqrt{\boldsymbol{2}\Delta P\boldsymbol{\rho}_{\boldsymbol{1}}}$$

*(Eq. 1)*

where:

- $\dot{m}$ is the fluid discharge mass flow rate
- $C$ is the discharge coefficient
- $\beta$ is the ratio of orifice to pipe internal diameter (*d/D* )
- $\varepsilon$ is the expansibility factor
- $d$ is the orifice diameter
- $\Delta P$ is the measured pressure difference across the orifice plate tappings
- $\rho_1$ is the upstream fluid density

For a given (hypothetically fixed) mass flow rate and a desired measured pressure difference, the orifice plate diameter *d* is the required unknown. It may be more convenient, therefore, to reorganise (Eq. 1) in terms of the measured pressure difference:

$$\Delta P = \frac{8\,\dot{m}^2\,(1-\beta^4)}{\rho\,(\pi\,d^2\,C\,\varepsilon)^2}$$

*(Eq. 2)*

Using the Flownex *Designer*, the orifice diameter *d* is varied until the required measured pressure difference ΔP is found for the given (fixed) mass flow rate $\dot{m}$.

Since the flow rate is given as a volume flow at normal conditions (NTP), the equivalent mass flow has to be determined since flow rates are specified in terms of mass in Flownex and not volume flows. Unfortunately, like standard conditions (STP), normal conditions are not universally taken at the same pressure and temperature. In this case study, normal conditions are taken to be at 0°C and 101.325 kPa-a. The easiest method for finding the mass flow rate equivalent to the specified NTP volume flow rate is by employing the *Designer*. But in order to allow Flownex to correctly understand NTP conditions, the ambient conditions in the *Flow Solver* settings has to be set correctly:

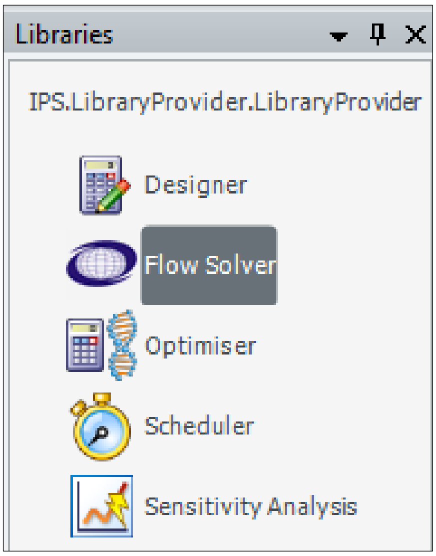



*Figure 2: Selecting the Flow Solver under the Solver/Utilities tab.*

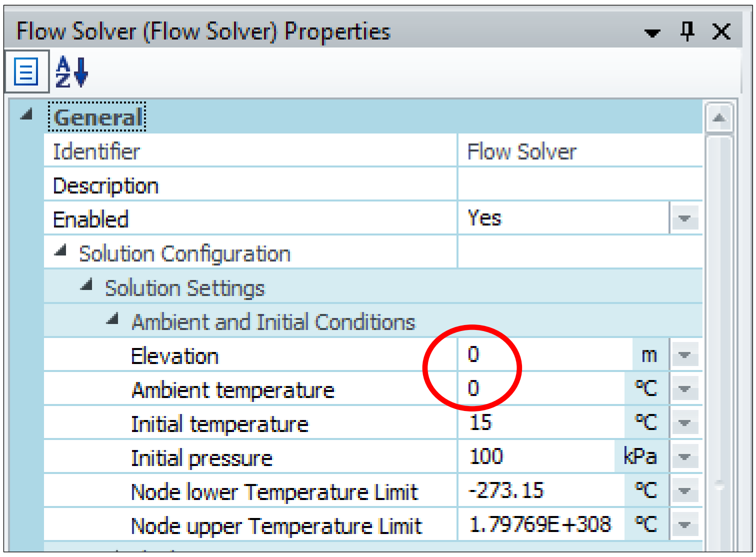



*Figure 3: Ambient Conditions: 0°C at 0 m Elevation (101.325 kPa).*

Ensure that the elevation as well as the ambient temperature is set to zero as indicated. Then set up a *Designer* configuration which searches the upstream pipe "*Volume flow based on ambient conditions*" for a value of 1500 m$^3$/h. The outlet boundary condition mass flow rate, which will be a negative value, is the independent variable in the search. The configuration for this case is shown in Figure 4. Running the *Designer* with this configuration set active will quickly find the equivalent mass flow rate and assign it to the outlet boundary.

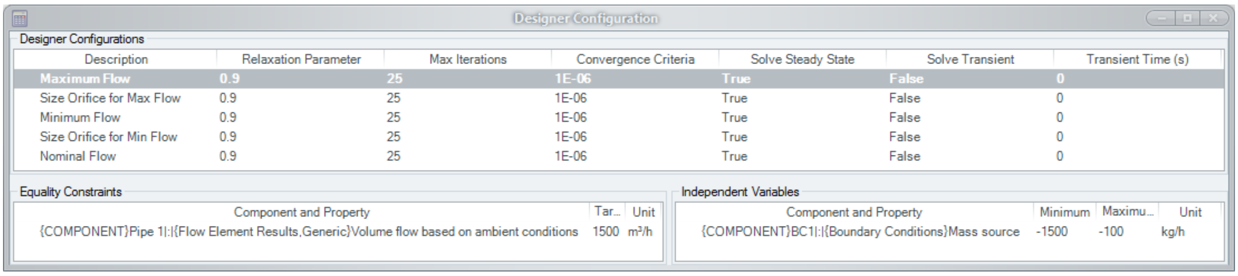



*Figure 4: Designer Configuration to Find the Volume Flow Rate at NTP.*

Alternatively, the inlet boundary pressure and temperature values could also be temporarily set to normal condition values (0°C and 101.325 kPa-a) and the actual volume flow used in the Designer search.

With the mass flow rate known, the *Designer* is used again, this time to find the required orifice diameter whilst keeping the mass flow rate constant. The orifice plate result "*Measured static pressure difference*" is the equality constraint whilst the orifice plate input "*Orifice diameter*" forms the independent variable. Running the *Designer* with this configuration set as active arrives at the results shown in Figure 1 where an orifice diameter of 64.170 mm is calculated. Obviously, the actual orifice diameter will be chosen to represent a more standard size such as 64.0 mm or 64.5 mm. However, before a final decision can be taken, the orifice plate measured pressure drop has to be checked against the minimum flow case. Once again, the *Designer* is used to find the minimum flow.

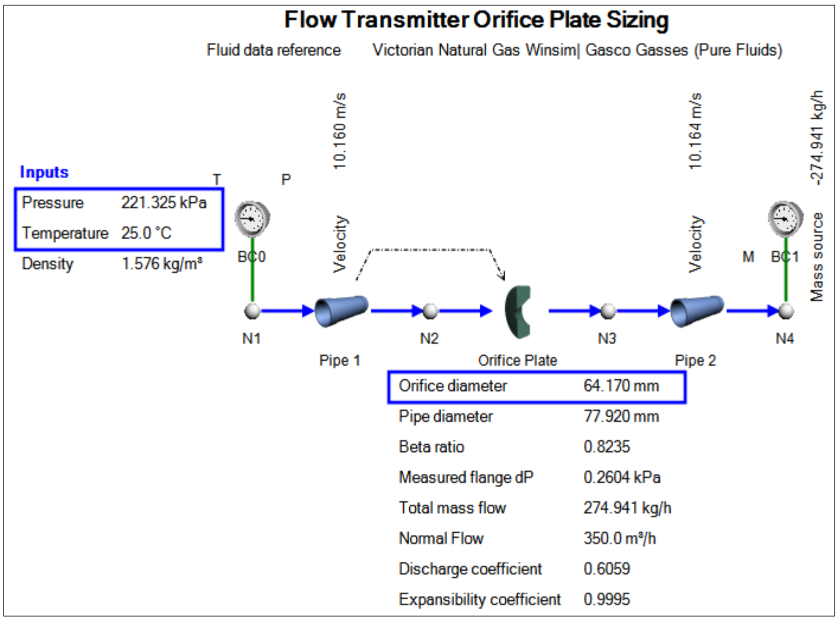



*Figure 5: Testing the Low Flow Case (0.3 kPa measured).*

As shown, the measured pressure difference across the orifice plate is only 0.26 kPa which is significantly less than the target value of 0.5 kPa. If accuracy at low flow is important, a pressure transmitter with the capability of accurately measuring a pressure difference range from 0.2 kPa to 5 kPa must be sourced.

Alternatively, if the pressure transmitter can handle pressure differences higher than 5 kPa, the orifice may be sized for the minimum flow case instead and tested against the maximum flow case. Using *Designer*, an orifice of 57.851 mm would be required to produce a 0.5 kPa pressure difference at 350 $Nm^3/h$ as shown in Figure 6. This orifice size, however, would produce a 9.7 kPa pressure difference at maximum flow as shown in Figure 7, so the pressure transmitter must be able to measure the range 0.5 kPa to 10 kPa.

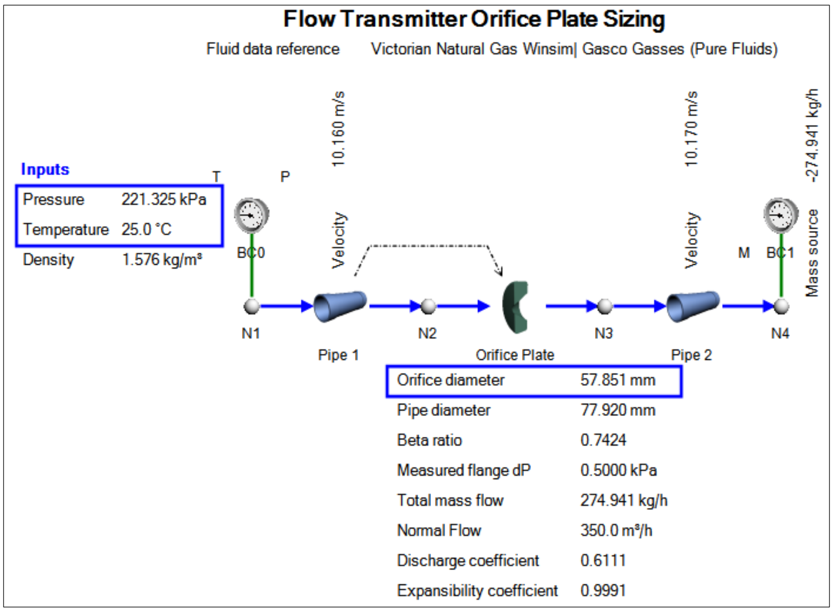



*Figure 6: Sizing the Orifice for the Minimum Flow Case (0.5 kPa measured).*

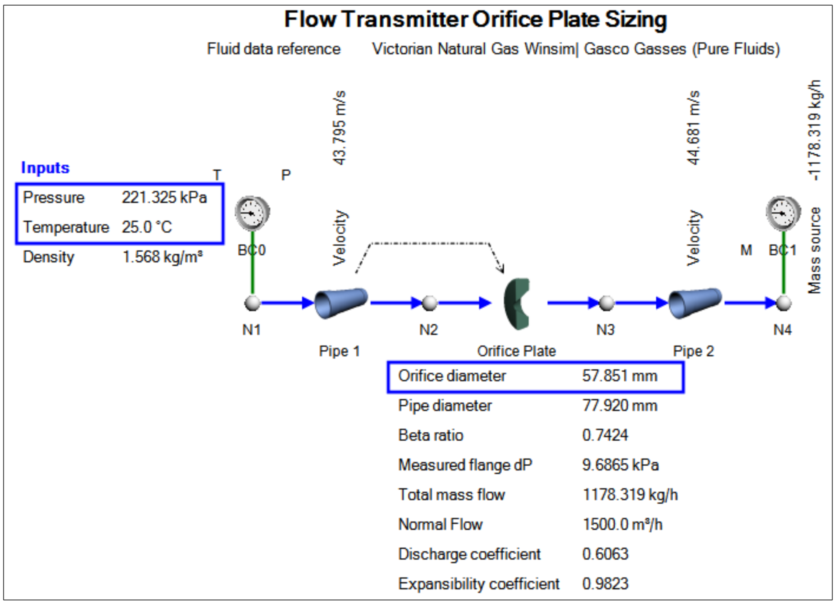



*Figure 7: Testing the Orifice for the Maximum Flow Case (9.7 kPa measured).*

Assuming that a more standard size such as a 57.5 mm orifice plate will be manufactured instead of the calculated 57.851 mm produced by the Flownex *Designer*, the calculations at minimum, normal and maximum flow must be repeated. The results are given in the following table.

Table 1: Flownex Results for the Actual Orifice Size of 57.5 mm.

| | Unit | Minimum Flow | Nominal Flow | Maximum Flow |
|---|---|---|---|---|
| Normal Flow Rate | $Nm^3/hr$ | 350 | 1200 | 1500 |
| Mass Flow Rate | kg/hr | 274.9 | 942.7 | 1178.3 |
| Supply Pressure | kPa-a | 221.325 | 221.325 | 221.325 |
| Supply Temperature | °C | 25 | 25 | 25 |
| Orifice Diameter | mm | 57.5 | 57.5 | 57.5 |
| Orifice Beta Ratio | - | 0.7379 | 0.7379 | 0.7379 |
| Measured Flange dP | kPa | 0.5175 | 6.3160 | 10.0331 |

For the purposes of this case study, it is assumed that a pressure transducer and transmitter with measurements ranging from 0.5 kPa to 10 kPa are available. This concludes the sizing phase of the flow meter and leaves the uncertainty analysis to be performed.

## Estimation of the Flow Meter Measurement Uncertainty

When performing an uncertainty analysis the *real* value of a property is rarely known since it is deduced from the *measured* value and estimates of the *uncertainties* of measurement. For a given mass flow rate $\dot{m}$ in (Eq. 2), $\Delta P$ may be calculated to provide the theoretical *(real)* orifice plate pressure difference value, however, several of the terms in the equation contain uncertainties. Therefore, the actual measured pressure difference would also contain some uncertainties which will be interpreted as uncertainties in the flow rate $\dot{m}$ by the transmitter.

In this case study, the approach to uncertainty analysis is that the *real* values (flow rates) have been nominated at the start and are therefore known. The maximum flow rate is to be 1500 $Nm^3/hr$, the nominal flow rate 1200 $Nm^3$/hr and the minimum flow rate is 350 $Nm^3$/hr. What is *uncertain* is the accuracy with which the orifice plate and transmitter combination is able to *measure* these *real* flow rates. Therefore in this case study, the aim is to determine the difference between the *real* and *measured* values. This difference is the *uncertainty* of measurement.

The case study focuses on two ways to determine this *uncertainty*:

- A mathematical approach where the governing equations are analyzed and an overall flow measurement uncertainty established. This approach demonstrates how quickly pure mathematical modeling will grow beyond the scope of most design engineers.
- Performing a Monte Carlo analysis using Flownex's *Sensitivity Analysis* feature will demonstrate the simplicity of conducting complex uncertainty analyses which are still within the capability of any engineer equipped with Flownex.

### *Mathematical Uncertainty Estimation*

With a known flow rate the theoretically measured pressure difference may be determined from (Eq. 2). However, the actual installed orifice plate flow meter may measure slightly different pressure differences to what theory suggests because of the following:

- The gas supply pressure may change.
- The gas supply temperature may change.
- The orifice plate has a manufacturing tolerance which will influence measured results.
- According to ISO 5167 there is some uncertainty in (Eq. 2).
- The gas composition may change.
- The pressure transmitter has less than perfect accuracy, stability and repeatability.

The influence of all of these variables combined or in isolation on the flow measurement uncertainty of the orifice plate may be estimated from first principles using the well-known root sum squared approach:

$$\boldsymbol{u_{\Delta P} = \sqrt{u_P^2 + u_T^2 + u_d^2 + u_C^2 + u_G^2 + u_X^2 \ldots}} \qquad \textit{(Eq. 3)}$$

where $u_{\Delta P}$ is the combined resultant measured pressure difference uncertainty $\delta(\Delta P)$ of all the independent uncertainties
$u_P$ is the pressure-related uncertainty $\delta(\Delta P)_P$
$u_T$ is the temperature-related uncertainty $\delta(\Delta P)_T$
$u_d$ is the orifice plate manufacturing tolerance-related uncertainty $\delta(\Delta P)_d$
$u_C$ is the ISO 5167 discharge coefficient uncertainty $\delta(\Delta P)_C$ (Eq. 2)
$u_G$ is the gas composition-related uncertainty $\delta(\Delta P)_G$
$u_X$ is the pressure transducer and transmitter-related uncertainty $\delta(\Delta P)_X$

To be able to calculate each of the terms above, (Eq. 2) has to be evaluated in terms of changes of each of the terms in (Eq. 3). However, as can be seen, (Eq. 2) does not contain any direct pressure or temperature terms, but these uncertainties may be accounted for indirectly via the density. Similarly, changes in the gas composition could also be reflected as changes in density. The transducer/transmitter uncertainties are given by the manufacturer and can be readily added to the above equation. Orifice plate diameter tolerances are also known.

The measured pressure difference uncertainty $\delta(\Delta P)$ caused by each of these variables $\varphi$ is established by determining the rate of change of the measured pressure difference with changes in each variable $\partial(\Delta P)/\partial\varphi$ and multiplying it by the uncertainty of that variable $\delta\varphi$. For the case at hand the resultant uncertainty then becomes:

$$\boldsymbol{u_{\Delta P}} = \sqrt{\left(\frac{\boldsymbol{\partial(\Delta P)}}{\boldsymbol{\partial\rho}}\boldsymbol{\delta\rho}\right)^{2} + \left(\frac{\boldsymbol{\partial(\Delta P)}}{\boldsymbol{\partial d}}\boldsymbol{\delta d}\right)^{2} + \left(\frac{\boldsymbol{\partial(\Delta P)}}{\boldsymbol{\partial C}}\boldsymbol{\delta C}\right)^{2} + \left(\frac{\boldsymbol{\partial(\Delta P)}}{\boldsymbol{\partial X}}\boldsymbol{\delta X}\right)^{2}}$$

*(Eq. 4)*

<u>Density Uncertainty:</u>

From (Eq. 2), the relationship between the measured pressure difference and each of the variables with uncertainties in question are as follows:

$$\frac{\partial(\Delta P)}{\partial\rho_1} = \frac{-8\,\dot{m}^2(1-\beta^4)}{(\pi\,d^2\,C\,\varepsilon)^2\,\rho_1^2}$$

*(Eq. 5)*

The term $\delta\rho$ must be calculated from the influence of the pressure and temperature on the density. This may be done from the ideal gas equation of state:

$$\rho_1 = \frac{P_1 M}{Z\bar{R}T_1}$$

where: $Z$ is the ideal gas compressibility factor
$M$ is the molar mass of the gas
$\bar{R}$ is the universal gas constant

Therefore:

$$\frac{\partial\rho_1}{\partial T} = \frac{P_1 M}{Z\bar{R}T_1^2}$$

and thus

$$\delta\rho_T = \frac{P_1 M}{Z\bar{R}T_1^2}\delta T$$

*(Eq. 6)*

Similarly:

$$\delta\rho_P = \frac{M}{Z\bar{R}T_1}\delta P$$

*(Eq. 7)*

Assume that one could simply represent the expected changes in gas composition in terms of density changes by calculating the density from changes in the gas molar mass:

$$\frac{\partial(\Delta P)}{\partial G} = \frac{\partial(\Delta P)}{\partial M} = \frac{\partial(\Delta P)}{\partial \rho_1}\frac{\partial \rho_1}{\partial M}$$

From the ideal gas law above:

$$\delta\rho_G = \frac{P_1}{Z\bar{R}T_1}\delta M$$

*(Eq. 8)*

Typical natural gas has a molar mass between 17 kg/kmol and 20 kg/kmol. For this specific case, the natural gas composition used has a molar mass of 17.55 kg/kmol. It would be reasonable to assume that the gas molar mass could vary between 17.35 kg/kmol and 17.75 kg/kmol, i.e.:

$$\delta M = 0.4\ kg/kmol$$

Combining (Eq. 6), (Eq. 7) and (Eq. 8):

$$\delta\rho = \frac{P_1 M}{Z\bar{R}T_1^2}\delta T + \frac{M}{Z\bar{R}T_1}\delta P + \frac{P_1}{Z\bar{R}T_1}\delta M = \frac{1}{Z\bar{R}T_1}\left(\frac{P_1 M\delta T}{T_1} + M\delta P + P_1\delta M\right)$$

*(Eq. 9)*

The square of the product of (Eq. 5) and (Eq. 9) represents the first term in (Eq. 4).

<u>Orifice Diameter Uncertainty:</u>

Since *$\beta$ = d / D*

$$\frac{\partial(\Delta P)}{\partial d} = \frac{-32\ \dot{m}^2}{(\pi\ C\ \varepsilon)^2\ \rho_1 d^5}$$

*(Eq. 10)*

The term $\delta d$ relates to the uncertainty in the orifice diameter:

$$\delta d = 0.2\ mm$$

*(Eq. 11)*

The square of the product of (Eq. 10) and $\delta d$ represents the second term in (Eq. 4).

<u>Orifice Plate ISO Uncertainty:</u>

According to ISO 5167 an uncertainty exists for the discharge coefficient *C* in (Eq. 2). Therefore:

$$\frac{\partial(\Delta P)}{\partial C} = \frac{-16\ \dot{m}^2(1-\beta^4)}{(\pi\ d^2\varepsilon)^2\ \rho_1 C^3}$$

*(Eq. 12)*

The discharge coefficient uncertainty as given by ISO 5167 for orifice plate beta ratios between 0.6 and 0.75 is:

$$\delta C = (1.667\beta - 0.5)\%$$

*(Eq. 13)*

For the case at hand the beta ratio (orifice to pipe internal diameter) is:

$$\beta = 0.7379$$

Since the diameter is larger than 71.12 mm and the Reynolds Number ($Re_D$) is larger than 10 000, the additional uncertainty correction terms specified by ISO 5167 are not applicable. Therefore:

$$\delta C = 0.73\%$$

*(Eq. 14)*

Note however that (Eq. 13) may not be valid for other beta ratios, and that other uncertainty terms may be applicable at other Reynolds numbers.

The square of the product of (Eq. 7) and $\delta C$ represents the third term in (Eq. 4).

<u>Transducer and Transmitter Uncertainty:</u>

The transducer/transmitter uncertainty is given by the manufacturer directly in terms of a measured pressure difference uncertainty:

$$u_X = \left(\frac{\partial(\Delta P)}{\partial X}\delta X\right) = \frac{(2 \times 0.075 + \times\ 2 \times 0.01)}{100} \cdot \Delta P = 0.0017\Delta P$$

*(Eq. 15)*

Total Combined Uncertainty:

The above uncertainty terms must now be substituted into (Eq. 4):

$$u_{\Delta P} = \sqrt{\begin{array}{c}\left(\dfrac{-8\,\dot{m}^2(1-\beta^4)}{(\pi\,d^2\,C\,\varepsilon)^2\,\rho_1^2}\left(\dfrac{1}{Z\bar{R}T_1}\left(\dfrac{P_1 M\delta T}{T_1}+M\delta P+P_1\delta M\right)\right)\right)^2+\\ \left(\dfrac{-32\,\dot{m}^2}{(\pi\,C\,\varepsilon)^2\,\rho_1 d^5}\delta d\right)^2+\\ \left(\dfrac{-16\,\dot{m}^2(1-\beta^4)}{(\pi\,d^2\varepsilon)^2\,\rho_1 C^3}\delta C\right)^2+\\ (0.0017\Delta P)^2\end{array}}$$

*(Eq. 16)*

However, this equation is still incomplete. Referring to (Eq. 2), the expansibility coefficient $\varepsilon$ is a function of the upstream and downstream pressures $P_1$ and $P_2$ and the upstream density $\rho_1$ and should also have been included in the first two terms of (Eq. 16):

$$\varepsilon = 1-(0.351+0.256\beta^4+0.93\beta^8)\left[1-\left(\frac{P_2}{P_1}\right)^{1/k}\right]$$

where

$P_1$ and $P_2$ are the upstream and downstream pressures respectively
$k$ is the specific heat ratio

There are other factors too that have been omitted in the above discussion. It should be clear by now that even for this relatively simple problem, this approach would quickly escalate beyond the mathematical capabilities of most engineers in industry, including this author. Fortunately Flownex has the capability to perform the same uncertainty analysis numerically in a way that is easy to use for anyone familiar with Flownex.

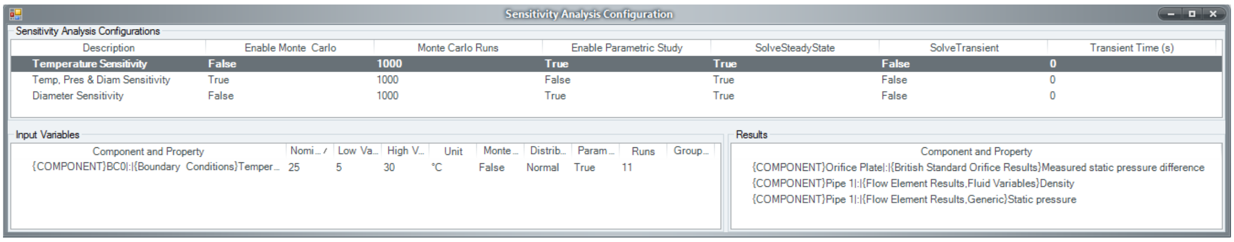
## *Estimation of the Flow Meter Measurement Uncertainty using Flownex's Sensitivity Analysis Feature*

Flownex's *Sensitivity Analysis* feature may be used to perform uncertainty analyses following two slightly different approaches:

- A series of parametric analyses may be performed, each analysis varying only one parameter such as those represented by (Eq. 3), and then combining their overall influence by using (Eq. 3) or (Eq. 16).
- Alternatively, all fluid flow related variables may be combined into a single Monte Carlo analysis, resulting in a single calculated uncertainty for these variables.

### Parametric Study

As an example of the first approach, a parametric study of the influence of temperature alone on the flow measurement is demonstrated. Since the mass flow is kept constant, the density and therefore the actual volume flow rate has to vary in concurrence with the temperature. Variations in the volume flow rate will cause variations in measured pressure difference which the flow transmitter will translate as changes in flow rate. As shown in Figure 8, the temperature is varied between 5°C and 30°C with 25°C being the nominal value.



Sensitivity Analysis Configuration

Sensitivity Analysis Configurations

| Description | Enable Monte Carlo | Monte Carlo Runs | Enable Parametric Study | SolveSteadyState | SolveTransient | Transient Time (s) |
|---|---|---|---|---|---|---|
| **Temperature Sensitivity** | **False** | **1000** | **True** | **True** | **False** | **0** |
| Temp, Pres & Diam Sensitivity | True | 1000 | False | True | False | 0 |
| Diameter Sensitivity | False | 1000 | True | True | False | 0 |

Input Variables

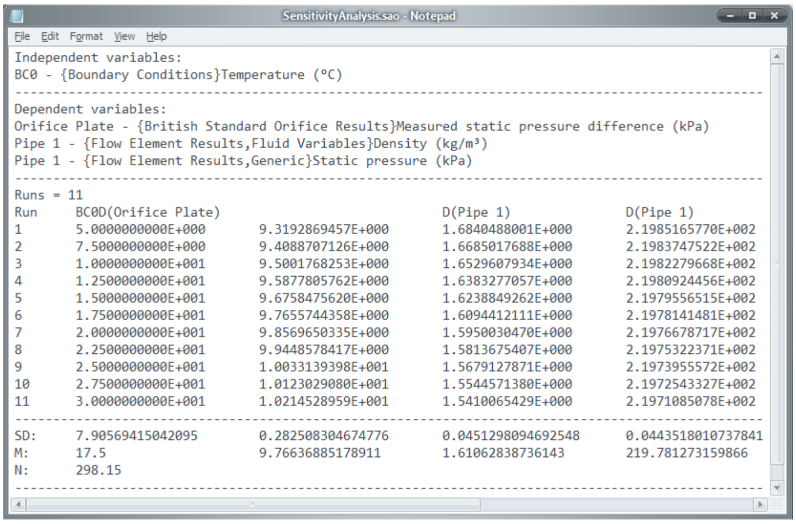
| Component and Property | Nomi... | Low Va... | High V... | Unit | Monte... | Distrib... | Param... | Runs | Group... |
|---|---|---|---|---|---|---|---|---|---|
| {COMPONENT}BC0\|:\|{Boundary Conditions}Temper... | 25 | 5 | 30 | °C | False | Normal | True | 11 | |

Results

| Component and Property |
|---|
| {COMPONENT}Orifice Plate\|:\|{British Standard Orifice Results}Measured static pressure difference |
| {COMPONENT}Pipe 1\|:\|{Flow Element Results,Fluid Variables}Density |
| {COMPONENT}Pipe 1\|:\|{Flow Element Results,Generic}Static pressure |

*Figure 8: Flownex Sensitivity Analysis Configuration for temperature uncertainty.*

Running this configuration in the *Sensitivity Analysis* tool at the maximum flow rate produces the following results:



```
SensitivityAnalysis.sao - Notepad
File  Edit  Format  View  Help
Independent variables:
BC0 - {Boundary Conditions}Temperature (°C)
--------------------------------------------------------------------------------------------------
Dependent variables:
Orifice Plate - {British Standard Orifice Results}Measured static pressure difference (kPa)
Pipe 1 - {Flow Element Results,Fluid Variables}Density (kg/m³)
Pipe 1 - {Flow Element Results,Generic}Static pressure (kPa)
--------------------------------------------------------------------------------------------------
Runs = 11
Run     BC0D(Orifice Plate)                          D(Pipe 1)              D(Pipe 1)
1       5.0000000000E+000       9.3192869457E+000    1.6840488001E+000      2.1985165770E+002
2       7.5000000000E+000       9.4088707126E+000    1.6685017688E+000      2.1983747522E+002
3       1.0000000000E+001       9.5001768253E+000    1.6529607934E+000      2.1982279668E+002
4       1.2500000000E+001       9.5877805762E+000    1.6383277057E+000      2.1980924456E+002
5       1.5000000000E+001       9.6758475620E+000    1.6238849262E+000      2.1979556515E+002
6       1.7500000000E+001       9.7655744358E+000    1.6094412111E+000      2.1978141481E+002
7       2.0000000000E+001       9.8569650335E+000    1.5950030470E+000      2.1976678717E+002
8       2.2500000000E+001       9.9448578417E+000    1.5813675407E+000      2.1975322371E+002
9       2.5000000000E+001       1.0033139398E+001    1.5679127871E+000      2.1973955572E+002
10      2.7500000000E+001       1.0123029080E+001    1.5544571380E+000      2.1972543327E+002
11      3.0000000000E+001       1.0214528959E+001    1.5410065429E+000      2.1971085078E+002
--------------------------------------------------------------------------------------------------
SD:     7.90569415042095        0.282508304674776    0.0451298094692548     0.0443518010737841
M:      17.5                    9.76636885178911     1.61062838736143       219.781273159866
N:      298.15
--------------------------------------------------------------------------------------------------
```

*Figure 9: Temperature Parametric Study at Maximum Flow.*

The orifice plate is supposed to measure 10.0331 kPa as discussed earlier, however, the values range from 9.319 kPa at 5°C to 10.215 kPa at 30°C which represents a total variation of:

$$u_\rho = u_T = 0.896\ kPa$$

Comparing this result to the mathematical approach given by (Eq. 5):

$$u_\rho = \frac{\partial(\Delta P)}{\partial \rho_1}\delta\rho = \frac{-8\,\dot{m}^2(1-\beta^4)}{(\pi\ d^2\ C\ \varepsilon)^2\ \rho_1^2}\delta\rho$$

$$= \frac{-8(0.3273kg/s)^2 \times (1-0.7379^4)}{(\pi(0.0575m)^2 \times 0.6065 \times 0.9818)^2 \times (1.568kg/m^3)^2} \times (0.143kg/m^3)$$

$$= -0.915\ kPa$$

Even though these results differ by only 19 Pa, it should be noted that small variations in the expansibility coefficient and discharge coefficient will also result from changes in temperature since the volume flow rate will be changing. These have not been accounted for in the above equation. This exercise must now be repeated for each variable associated with an uncertainty and for each flow rate of interest.

## Monte Carlo Analysis

However, all of these influences will occur simultaneously, and this is where Flownex's *Monte Carlo Analysis* feature comes into its own. For the case at hand, the Monte Carlo configuration setup is as follows:

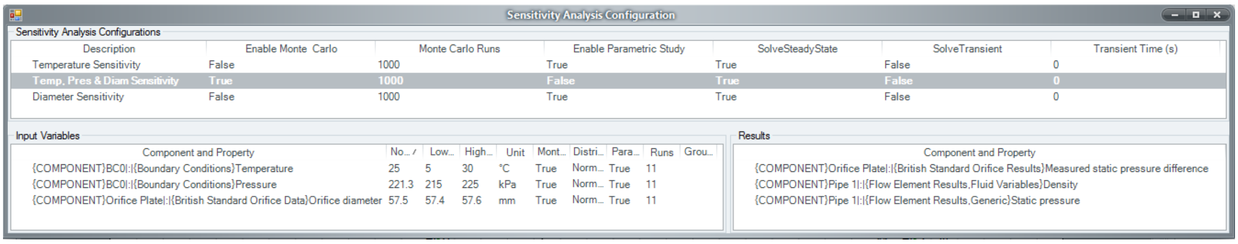



*Figure 10: Monte Carlo Analysis Configuration for Temperature, Pressure and Orifice Diameter.*

As before, the temperature is varied between 5°C and 30°C, the pressure is varied between 215 kPa-a and 225 kPa-a and the orifice diameter manufacturing tolerance is ±0.1 mm. For the purposes of demonstration, the Monte Carlo runs have been set to 1000 and could be set higher. The inputs are summarized in the following table:

Table 2: Flownex Inputs for the Monte Carlo Analysis.

| | Unit | Fixed Values |
|---|---|---|
| Normal Flow Rates | $Nm^3/hr$ | 350, 1200, 1500 |
| Upstream Pressure | kPa | 221.3 |
| Upstream Temperature | °C | 25 |
| Orifice Diameter | mm | 57.5 |
| | | Uncertainties |
| Upstream Pressure Variation $u_p$ | kPa | +3.7 -6.3 |
| Upstream Temperature Variation $u_T$ | °C | +5 -20 |
| Orifice Diameter Variation $u_d$ | mm | ±0.1 |
| Transmitter Uncertainty $u_x$ | % Span | ±0.085 |
| Discharge Coefficient Uncertainty $u_C$ | % Flow | 0.73 |

Even though the uncertainties $u_P$, $u_T$ and $u_d$ are supplied as three separate inputs, Flownex will calculate a single combined measured pressure difference uncertainty $u_{PTd}$. Note that the transmitter and discharge coefficient uncertainties cannot be added to the Flownex Monte Carlo run as they are not flow property related. Hence they must be added mathematically at a later stage. Also note that the uncertainty in gas composition has been ignored for the sake of simplicity.

Running this sensitivity analysis configuration allows Flownex to vary all three of these variables simultaneously, searching for the combination that would result in the maximum variation. The results are shown in the following figure:

```
Monte Carlo Analysis Results 1500.txt - Notepad
File  Edit  Format  View  Help
Independent variables:
BC0 - {Boundary Conditions}Temperature (°C)
BC0 - {Boundary Conditions}Pressure (kPa)
Orifice Plate - {British Standard Orifice Data}Orifice diameter (mm)
------------------------------------------------------------------------------------------------
Dependent variables:
Orifice Plate - {British Standard Orifice Results}Measured static pressure difference (kPa)
Pipe 1 - {Flow Element Results,Fluid Variables}Density (kg/m³)
Pipe 1 - {Flow Element Results,Generic}Static pressure (kPa)
------------------------------------------------------------------------------------------------
1       2.1327864341E+001       2.1708368830E+002       5.7525525690E+001       1.0090908443E+001
2       1.7713447124E+001       2.1905485292E+002       5.7475682196E+001       9.9085252259E+000
3       1.8764455712E+001       2.1921636524E+002       5.7519642475E+001       9.8956722826E+000
4       8.2552275718E+000       2.1916949971E+002       5.7498914550E+001       9.5389582807E+000
5       2.5437128437E+001       2.1932495025E+002       5.7488020417E+001       1.0161601543E+001
...
997     1.9964546025E+001       2.2149813558E+002       5.7561515168E+001       9.7867901483E+000
998     2.2979766183E+001       2.2062209354E+002       5.7552694146E+001       9.9439021389E+000
999     1.2815209477E+001       2.1756253857E+002       5.7512661746E+001       9.7685705430E+000
1000    2.8973396918E+001       2.2126181073E+002       5.7431072976E+001       1.0250475066E+001
------------------------------------------------------------------------------------------------
SD:     5.43249248724332        2.18834718439881        0.0435337454446798      0.223871609365947
M:      17.5520804045431        220.042358818998        57.5002217172877        9.83158410956998
N:      298.15  221300  0.0575  1.0250475066E+001       1.5460766900E+000       2.1965322761E+002
```

*Figure 11: Monte Carlo Analysis Results for Temperature, Pressure and Orifice Diameter.*

These results have been copied into Excel and the minimum and maximum measured pressure difference values obtained. For this case, the maximum combined uncertainty for pressure, temperature and orifice diameter is:

$$u_{PTd} = 1.337\ kPa$$

The other uncertainties $u_C$ and $u_X$ must be evaluated mathematically and then added to $u_{PTd}$ using (Eq. 3):

$$u_{\Delta P} = \sqrt{u_{PTd}^2 + u_C^2 + u_X^2}$$

From (Eq. 12) and (Eq. 14):

$$u_C = 0.242\ kPa$$

As given by the manufacturer in Table 2:

$$u_X = 0.0017 \times 10.0331\ kPa = 0.0171\ kPa$$

Therefore:

$$u_{\Delta P(max\ flow)} = \sqrt{1.337^2 + 0.242^2 + 0.0171^2}\ kPa$$

$$= 1.359\ kPa$$
$= 13.54\%$ of full scale

According to (Eq. 1) the uncertainty in measured pressure difference $u_{\Delta P}$ relates to an uncertainty in mass flow rate $u_{\dot{m}}$ as follows:

$$u_{\dot{m}} \propto \sqrt{u_{\Delta P}}$$

Therefore, for the maximum flow case:

$u_{\dot{m}\ max} = 3.68\%$ of full scale

Similarly, a Monte Carlo analysis must also be performed at the minimum and nominal flow cases. The results are summarized in the following table:

Table 3: Flownex Results for the Monte Carlo Analysis.

| | Unit | Min Flow | Nominal Flow | Max Flow |
|---|---|---|---|---|
| Normal flow rate | $Nm^3/hr$ | 350 | 1200 | 1500 |
| Mass flow rate | kg/hr | 274.9 | 942.7 | 1178.3 |
| Theoretical measured pressure difference $\Delta P$ | kPa | 0.5175 | 6.316 | 10.033 |
| Measured pressure difference uncertainty $u_{PTd}$ | kPa | 0.0596 | 0.8565 | 1.3373 |
| Transmitter uncertainty $u_X$ | kPa | 0.0032 | 0.0109 | 0.0137 |
| Discharge coefficient uncertainty $u_C$ | kPa | 0.0132 | 0.1549 | 0.2420 |
| Total measured pressure difference uncertainty $u_{\Delta P}$ | kPa | 0.0611 | 0.8705 | 1.3591 |
| | % $\Delta P$ | 11.8 | 13.8 | 13.6 |
| Mass flow rate uncertainty $u_{\dot{m}}$ | kg/hr | 9.4 | 35.0 | 43.4 |
| | % Mass flow | 3.4 | 3.7 | 3.7 |

As shown in the results, the mass flow uncertainties are approximately 3.7%. If this level of accuracy is unacceptable, temperature and pressure correction may be implemented by adding pressure and temperature sensors to the upstream flow and adding appropriate logic to the transmitter to compensate for changes in the density.

Furthermore, orifice diameter manufacturing tolerances may also be improved. The results show that the bulk of the uncertainty in this case originates from the pressure, temperature and orifice diameter uncertainty $u_{PTd}$ – refer to Figure 12 below. A single parametric study on the influence of the manufacturing tolerance at maximum flow highlights that the contribution of the orifice tolerance to the overall uncertainty is approximately 14.8% of the total measured pressure difference uncertainty $u_{\Delta P}$. It can be concluded therefore that the bulk of the uncertainty originates from pressure and temperature related uncertainties.

Figure 12 shows the comparative uncertainties as a function of flow rate. As expected, the uncertainties increase with increasing flow rate. The transducer and transmitter related uncertainties are negligible in comparison, and the discharge coefficient uncertainty only contributes approximately 18% of the total. The bulk of the uncertainties are due to changes in pressure, temperature and the orifice plate diameter tolerance.

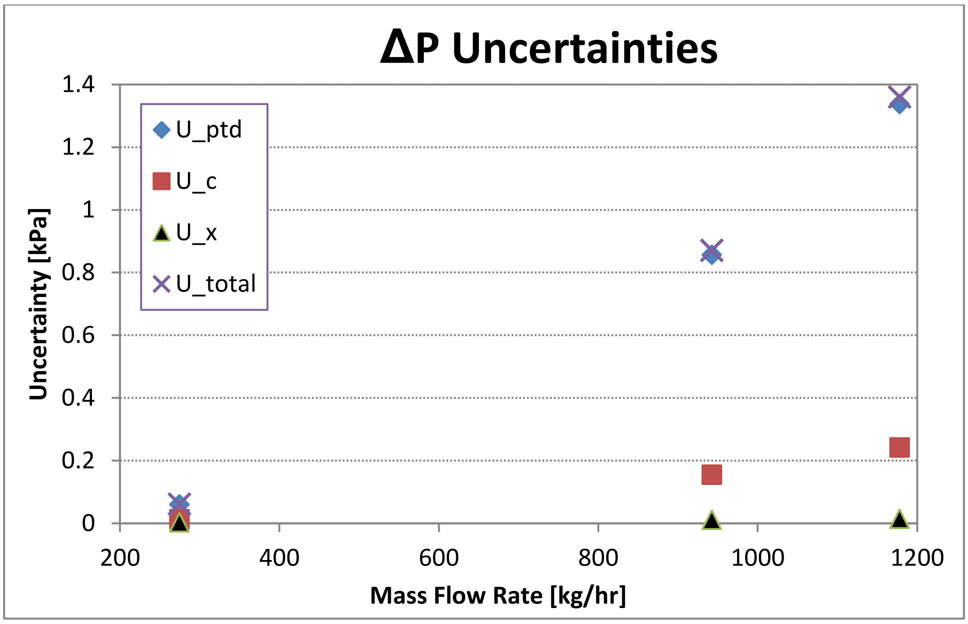



*Figure 12: Comparative Pressure Drop Uncertainties as a Function of Flow Rate.*

## SUMMARY

For the case studied, it is shown that the bulk of the uncertainty originates from pressure and temperature variations. If the calculated mass flow uncertainty of 3.7% is not acceptable, pressure and temperature correction has to be implemented in the transmitter. The orifice plate manufacturing tolerance contributes less to the overall uncertainty but is relatively simple to improve.

Not only is Flownex capable of designing large, sophisticated flow networks, but it is also able to focus on a single element of design such as a simple orifice plate and then analyse that element in incredible depth. This versatility of Flownex is unparalleled amongst similar products and must make Flownex an indispensable software tool amongst process design engineers.

## CASE STUDY FLOWNEX MODEL AVAILABILITY

The Flownex model discussed in this case study is available in the user project downloads area located at: http://www.flownex.com/projectlibrary